Modeling and Analysis of Missile Hitting Target in Dense Array Weapon System

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Abstract: The method of judging the target hit of the "dense array" projectile in the process of "dense array" firing simulation was studied. By studying the necessary conditions of hitting, the judgment process is limited to a small range near the target. By establishing the target equivalent body to simplify the shape of the target, the motion of the "dense array" projectile and the target in the ground coordinate system is transformed into the relative motion of the projectile in the projectile body coordinate system by using the kinematics principle, and the corresponding motion equation is established. By calculating the trajectory of the projectile, the hitting information is obtained, which makes the calculation amount enormous. The process of reduction and judgment is independent and suitable to be realized by MATLAB on computer. In addition to the hitting coordinates, the judgment method also gives the angle, which provides conditions for the judgment of hitting effectiveness.

1. Introduction

Since the late 1940s, the United States, the Soviet Union and Sweden began to develop anti-ship missiles on the basis of German V-1 missiles. Compared with traditional weapons, anti-ship missiles have many distinct characteristics, such as long range, high precision and great destructive power.

The rise of anti-ship missiles has seriously threatened the safety of surface ships. Although shipborne electronic countermeasure system and medium and short range air defense missile system are effective means of "decoy" and intercepting anti-ship missiles, the increasingly advanced anti-ship missiles still lack penetration and "leak net". How to intercept, kill and destroy these penetrating and "leaking" anti-ship missiles at last has attracted great attention of navies all over the world [1-5].

The "Dense Array" system is a short-range defense naval gun system developed and manufactured by the United States. It has undergone three stages of development: Block 0 (basic type, service in 1978), Block 1 (improved type), Block 1A (service in 1988) and Block 1B (service in 1999). Now it is developing "sea" based on the "Dense Array" system. SeaRAM short range point defense system. The primary task of "dense array" is to deal with low altitude and high speed targets (such as anti-ship missiles), and with the help of other systems [6-12], it can also deal with high elevation dive targets and small surface targets. It consists of a Trinity (search radar, tracking radar, computer and six-barrel barrel gun), local console and remote control console.

2. Target Domain and Target Equivalents of Dense Array Shooting

Firstly, we introduce two concepts: target domain and target equivalent.

Target domain: The center of the target is the sphere, which contains the smallest sphere of the target. Let the radius of the sphere be zero.

Target Equivalent Body: The geometry obtained by splicing the target shape with three standard geometries, cuboid, sphere and cylinder, and several planes which are cut off from the sphere and cylinder, can only be connected or separated. In order to simplify the calculation, the axes of

cuboids and cylinders are required to be parallel to the axes of the target coordinate system. Several planes in the target equivalent body are used to cut the sphere and cylinder into desired shapes. These planes are parallel to one of the coordinate planes in the target coordinate system.

According to the vulnerability analysis of anti-ship missile, we can know that the wing, the secondary rudder and the tail of the missile are not the fatal parts. In the simulation, we focus on the situation of the missile body, but ignore these three parts, and analyze the impact of the dense array on the remaining equivalent body. At the same time, in order to simplify the model, the remaining cylindrical and hemispherical bodies can be equivalent to a cylindrical model with the length and radius of the missile, and the critical part of the anti-ship missile is approximately equivalent in the range part.

3. Necessary Conditions and Relative Motion Equation of "Dense Array" Projectile Hitting Target

At the time node in the simulation process, we assum $V_n = [v_{xn}, v_{yn}, v_{zn}]^T$ are the velocity vector in the coordinates of the "dense array" projectile in the ground coordinate system, and $\tilde{V}_n = [\tilde{v}_{xn}, \tilde{v}_{yn}, \tilde{v}_{zn}]^T$ are the velocity vector in the coordinates of the center of the anti-ship missile.

In each step of the simulation, the square of the distance between the projectile and the anti-ship missile is calculated.

$$S_n^2 = (x_n - \tilde{x}_n)^2 + (y_n - \tilde{y}_n)^2 + (z_n - \tilde{z}_n)^2$$
(1)

The sum vector inner product is

$$\Omega_n = (v_{xn} - \tilde{v}_{xn})(\tilde{x}_n - x_n) + (v_{yn} - \tilde{v}_{yn})(\tilde{y}_n - y_n) + (v_{zn} - \tilde{v}_{zn})(\tilde{z}_n - z_n)$$
(2)

Two time nodes in the simulation process are selected to satisfy the following requirements:

$$f = \max\left\{n : n \in N, S_n^2 \ge R^2, \Omega_n > 0\right\}$$
(3)

$$b = \min\{n : n \in N, S^{2}_{n} \ge R^{2}, \Omega_{n} < 0\}$$
(4)

Use the Coordinate transformation matrix to solve the position and speed vectors of missile and projectile as following:

$$[x_{mf}, y_{mf}, z_{mf}]^{T} = \vec{L}(\gamma, \theta, \varphi) \left\{ [x_{f}, y_{f}, z_{f}]^{T} - [\widetilde{x}_{f}, \widetilde{y}_{f}, \widetilde{z}_{f}]^{T} \right\}$$
(5)

$$[x_{mb}, y_{mb}, z_{mb}]^{T} = \vec{L}(\gamma, \theta, \varphi) \left\{ [x_{b}, y_{b}, z_{b}]^{T} - [\widetilde{x}_{b}, \widetilde{y}_{b}, \widetilde{z}_{b}]^{T} \right\}$$
(6)

$$\widetilde{V}_{mf} = [\widetilde{v}_{xmf}, \widetilde{v}_{ymf}, \widetilde{v}_{zmf}]^{T} = \vec{L}(\gamma, \theta, \varphi) [\widetilde{v}_{xf}, \widetilde{v}_{yf}, \widetilde{v}_{zf}]^{T}$$
(7)

$$\widetilde{V}_{mb} = [\widetilde{v}_{xmb}, \widetilde{v}_{ymb}, \widetilde{v}_{zmb}]^{T} = \vec{L}(\gamma, \theta, \varphi) [\widetilde{v}_{xb}, \widetilde{v}_{yb}, \widetilde{v}_{zb}]^{T}$$
(8)

$$V_{mf} = [v_{xmf}, v_{ymf}, v_{zmf}]^{T} = \vec{L}(\gamma, \theta, \varphi) [v_{xf}, v_{yf}, v_{zf}]^{T}$$
(9)

$$V_{mb} = [v_{xmb}, v_{ymb}, v_{zmb}]^{T} = \vec{L}(\gamma, \theta, \varphi) [v_{xb}, v_{yb}, v_{zb}]^{T}$$
(10)

Then the relative velocity of the projectile "dense array" in the point A_f, A_b is calculated as

$$V_{rf} = \begin{bmatrix} v_{rxmf} \\ v_{rymf} \\ v_{rzmf} \end{bmatrix} = \begin{bmatrix} v_{xmf} \\ v_{ymf} \\ v_{zmf} \end{bmatrix} - \begin{bmatrix} \widetilde{v}_{xmf} \\ \widetilde{v}_{ymf} \\ \widetilde{v}_{zmf} \end{bmatrix}$$
(11)

$$V_{rb} = \begin{bmatrix} v_{rxmb} \\ v_{rymb} \\ v_{rzmb} \end{bmatrix} = \begin{bmatrix} v_{xmb} \\ v_{ymb} \\ v_{zmb} \end{bmatrix} - \begin{bmatrix} \widetilde{v}_{xmb} \\ \widetilde{v}_{ymb} \\ \widetilde{v}_{zmb} \end{bmatrix}$$
(12)

The time node t_f of the simulation process is regarded as the time zero of the relative motion of the projectile in the projectile coordinate system. During this time interval, the relative motion equation of the projectile in the projectile coordinate system can be used to simulate its motion. The relative motion equation is as follows:

$$\begin{cases} x_{t} = \frac{1}{2}a_{x} \cdot \delta t^{2} + v_{rxmf} \cdot \delta t + x_{mf} \\ y_{t} = \frac{1}{2}a_{y} \cdot \delta t^{2} + v_{rymf} \cdot \delta t + y_{mf} \\ z_{t} = \frac{1}{2}a_{z} \cdot \delta t^{2} + v_{rzmf} \cdot \delta t + z_{mf} \end{cases}$$
(13)

Where δt is the simulation step.

In missile body coordinate system, the range of target equivalent body is

$$(z - z_k)^2 + (y - y_k)^2 \le R^2 (-L/2 \le x \le L/2)$$
(14)

If the judgment point is within the range of the target equivalent body, the hit point (approximate value) can be obtained. At the same time, the simulation of relative motion and hit judgment of the "dense array" projectile is completed.

4. Conclusion

According to the conversion relationship between "ground coordinate system", "missile body coordinate system" and "transitional coordinate system", this paper establishes the judgment model of dense array firing and target hitting. Among them, the velocity and position information of anti-ship missile, target ship and "dense array" projectile in the ground coordinate system are easily obtained in the simulation model. Through the transitional coordinate system, the position and velocity information of the "dense array" projectile can be transformed into the coordinate system of the "missile body", and the target equivalent domain and the target equivalent body can be used to simplify the model of the anti-ship missile and judge the hit. It provides a basis for further judging the killing effect of "dense array" on anti-ship missiles.

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